*Journal of Engineering and Technology*
*Vol.4 No.2 July - December 2016*

# Simulated Annealing Algorithm for Vehicle Routing Problem with Transshipment

Sukanya Thonethong[#1], Jirachai Buddhakulsomsiri[#2]

[#]*Logistics and Supply Chain Systems Engineering*
*Sirindhorn International Institute of Technology, Thammasat University, Bangkok, Thailand*

[1]sukanya.thonethong@studentmail.siit.tu.ac.th
[2]jirachai@siit.tu.ac.th

***Abstract*— This paper considers the vehicle routing problem with transshipment. Customer demands are of two types: regular demands that are satisfied by the depot and transshipment demands of items from other customers. A simulated annealing (SA) algorithm to generate delivery routes in which both demands can be met in the same delivery routes is developed. The objective is to minimize the transportation cost. Preliminary results from testing the algorithm using numerical examples shows that allowing transshipment incurs additional cost of 0.48%-1.68%, depending on the level of transshipment demand, with the benefit of reducing delivery lead time.**

***Keywords*— Vehicle Routing Problem, Transshipment, Simulated Annealing**

## I. INTRODUCTION

Dantzig and Ramser [1] firstly introduced the Vehicle Routing Problem (VRP). The problem was modelled after a routing optimization problem for petrol deliveries by truck.The objective is to find the optimal set of routes for a fleet of vehicles to perform deliver services to a set of customer so as to minimize the total transportation cost. Since then, numerous research studies have been conducted to solve the VRP using heuristics and exact algorithms. There are also many variants of the VRP, such as VRP with time windows (VRPTW), VRP with pickup and delivery (VRPPD), etc.

This paper presents one of the variants of VRP, so-called, vehicle routing problem with transshipment (VRPT) in this paper. The motivation for the VRPT is from a real world problem found in one of the largest retail chains in Thailand. In this problem, a depot exists to satisfy daily demand from many retail stores, all of which are in the same retail chain under a single ownership. The retail chain offers products that are both continuously stocked and seasonal products. The focus is on the seasonal products, which are usually ordered once a year from both domestics and international suppliers.

These items arrive before the beginning of the selling season, and are kept at the depot, and the retail stores would order these items according to the store's projected sale figure.

By the middle of a season, all inventories of a seasonal item would be ordered and kept at the stores, and the depot would no longer have inventory of the item available. At this point, there are many occurrences when demands from end customers arise at a retail store that already has the items sold out, while the desired items are available at some other retail stores. The current practice of the company is as follows: (1) The drop-off store that need the items would issue a request to the depot. (2) Delivery truck that visits another pick-up store that has the item would pick up the item and bring it back to the depot. (3) The depot then sends the item to the drop-off store in the next delivery cycle. The process usually takes at least as long as the length of the delivery cycle. For example, the process takes at least one day if the deliveries to both the pick-up and drop-off stores are performed on a daily basis, or it takes at least two days, if delivery cycle is on alternate day basis.

The company is considering changing this process in order to satisfy the end customer demand in a shorter time. Specifically, the depot would like to design delivery routes that take into account the pick-up / drop-off demands between stores, which is called transshipment demand in this paper. The delivery routes that can satisfy the transshipment demand in addition to the regular demand from the depot must have the truck visits the pick-up store prior to the drop-off stores on the same route. In other words, the pick-up item from one store will be delivered to another store on the same trip.

Benefit from the same day delivery would give the retail stores a significant advantage in terms of customer service in the highly competitive retail business environment. The company would like to incorporate this change without having to incur too much additional delivery cost.

This paper presents an algorithm development for the VRPT that can generate good routing solution that allows both regular demand and transshipment demand deliveries on the same trip. The objective function is to minimize the total transportation cost. The algorithm is based on the well-known simulated annealing (SA) algorithm with solution generation mechanism that forces transshipment delivery.

The remainder of this paper is organized as follows. Section II provides a literature review, including VRP, solution techniques, and a VRP with transshipment center. The VRPT under consideration is described in Section III. The proposed algorithm is presented in Section IV. Then, Section V features the process of algorithm parameter tuning, followed by a numerical example in Section VI.

Finally, conclusion and future research direction are given in Section VII.

## II. LITERATURE REVIEW

### A. Vehicle Routing Problem

Due to the vast literature review on VRP, each variant of VRP will be briefly described, and followed by one selected recent research study for the problem. The classical vehicle routing problem (VRP) involves the problem to generate delivery route for a homogeneous fleet of vehicles and a single depot. Many variants of VRP have been studied. The capacitated vehicle routing problem (CVRP) is a VRP in which a homogenous fleet of delivery vehicles of same capacity must provide service to known customer demands.

The objective is to minimize the total cost, while the total demands delivered in each trip cannot exceed vehicle's capacity. Recent study on CVRP by Alipour [2] presents an algorithm based on distributed learning automata for solving CVRP.

Vehicle routing problem with backhauls (VRPB) is a VRP that considers both delivery (linehaul) and pickup (backhaul). VRPB assumes that all deliveries must be made on the route before pickups can be performed. Brandão [4] present a new tabu search algorithm that starting from pseudo-lower bounds for VRPB.

Vehicle routing problem with pick-up and delivery (VRPPD, sometimes denoted as PDP) is an extension of VRPB. Pure pickup or delivery only performs pickup or delivery in the routes. Mixed pickup and delivery has two types: (1) a route is not interspersed, which means the truck must finish all delivery demands before performing the pickup on the same route, and (2) interspersed route that mixes pickup and delivery on the same route. Another variant of the VRPPD is the VRP with simultaneous pick-up and delivery (VRPSPD), where delivery and pickup demands are required to be made simultaneously at each customer stop. Montané and Galvão [5] developed a tabu search algorithm to solve VRPSD.

Vehicle routing problem with time windows, or VRPTW, is a VRP where customers have specified time windows constraint in which the delivery must be made. Nagata et al. [6] present an effective memetic algorithm for the VRPTW.

Stochastic vehicle routing problem (SVRP) is a VRP where one or several components of the problem are random, such as stochastic demands, and stochastic travel times. Gendreau etal. [3] give a review of the scientific literature on SVRP.

### B. Solution Techniques for VRP

Solution techniques for VRP can be classified into three categories:

1) *Exact algorithms*: Exact algorithms, such as branch and bound algorithm and branch and cut algorithm, are methods that solve the problem to optimality. These algorithms have a limited size of VRP that they can solve due to the nonpolynomial nature of VRP.

2) *Heuristics*: Heuristics are methods that produce a good solution in a reasonable time. This solution obtained is neither guaranteed to be an optimal nor a feasible solution. These are the methods available to solve large-scale VRP effectively.

3) *Meta-heuristics*: Meta-heuristics are developed over the last two decades. They are similar to the heuristics, but have more sophisticated procedures that enable them to escape the local optimal. Examples are such as tabu search, genetic algorithm, and simulated annealing algorithm.

### C. Vehicle Routing Problem featuring Transshipment

One of the studies in the literature by Yang and Xiao [7] consider the transshipment characteristic of the problem. In their study, the VRP considers a multi-period single-product logistics system with transshipment centers. The transshipment centers can receive items from the depot and act as the second depot, after the transshipment of items from the main depot is made. In other words, the problem is similar to multi-depot problem, where additional depot is created from transshipment of items from the original depot. Although their problem is denoted with "transshipment," the problem is much different from the transshipment demand that is considered in this paper. To the best of our knowledge, as of this writing, there is no study that consider the transshipment demand in a VRP and mix them with the regular demand from the depot in the same way that we consider in this paper.

## III. VEHICLE ROUTING PROBLEM WITH TRANSSHIPMENT

Consider a VRP with one depot and a set of customers. These customers have two types of demand: regular demand that must be satisfied directly from the depot, and transshipment demand that can only be satisfied from inventory at another customer. The purpose is to generate good delivery routes for a fleet of homogenous capacitated trucks that allow deliveries of both demands on the same trip. The objective function is to minimize the total transportation cost.

A numerical example to describe an instance of the VRPT is presented next. Suppose there are 10 customers that must be served by depot. The depot has two delivery vehicles of same type and capacity. Each customer has daily demand from the depot that must be satisfied. The depot has no inventory leftover to satisfy the demand of a particular item, requested by Customer 5. However, this item is available at Customer 3. A VRPT solution for the two trucks is as illustrated in Fig. 1-2.The first route is the one that contains both Customer 3 and Customer 5, with Customer 3 being visited first, which enables delivery of the transshipment demand from Customer 3 to Customer 5.

Two important benefits from extending to VRPT is the reduction in both the lead time to deliver the item and the carryover demand from day-to-day. This is because without allowing transshipment demand to be delivered on the same trip, the truck would have to pick-up the item from Customer 3, bring it back to the depot, and deliver to Customer 5 on the next delivery cycle.

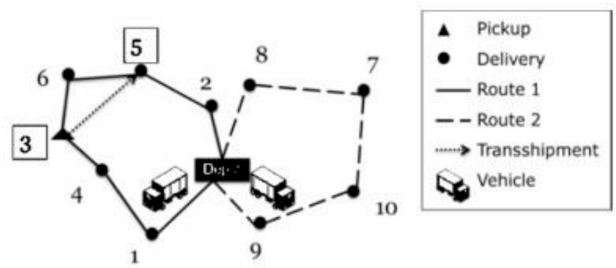


Fig. 1 An illustration of the vehicle routing with transshipment (VRPT)

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 4 | 3 | 6 | 5 | 2 | 0 | 0 | 9 | 10 | 7 | 8 | 0 |
| Route 1 | | | | | | | | Route 2 | | | | | |

Fig. 2 Presents the VRPT solution representation, delivery routes Route 1 = 0-1-4-3-6-5-2-0, Route 2 = 0-9-10-7-8-0

## IV. PROPOSED ALGORITHMS

The algorithm developed for the VRPT is based on the well-known simulated annealing (SA) algorithm, first introduced by Kirkpatrick et al. [8]. In general, SA is a meta-heuristic featuring a local search based on the concept of metal annealing process. Numerous research studies have used SA to solve many combinatorial optimization problems effectively.

The developed algorithm begins by generating an initial solution, and performing a local search within neighbor solutions. If the best neighbour solution improves the objective function, then the solution is accepted and replaces the current solution; otherwise, the worse solution may be accepted with a probability in order to escape the local minimum. This probability is computed from the Boltzmann function, $\exp(-\Delta\, kT)$, which contains three components: (1) the difference, Δ, between the current and the new solution, (2) a constant, $k$, and (3) a temperature, $T$.

At the beginning of the search, the temperature is set to a high initial temperature, $T_s$, which makes it easier to accept a worse solution. The algorithm would continue to perform a local search until it reaches a specified number of iterations. Then, the temperature is reduced by a cooling rate, α, and the local search resumes. The search is repeated until the temperature falls below a threshold, $T_F$, and the algorithm terminates and the best solution is reported.

The algorithm uses the following notation:

$T_s$ = Starting temperature
$T_f$ = Final temperature
α = Cooling rate
$S$ = Current solution
$S_b$ = Best neighbor solution found from the local search
$N$ = Number of iterations in each temperature

The algorithm can be described as follows:

Step 1: Set the algorithm parameters, $T_s$, $T_f$, $\alpha$, $N$, and initialize $T = T_s$.

Step 2: Generate an initial solution and keep it as the current solution S and the best solution $S_b$.

Step 3: Perform a local search in the neighbourhood of the current solution $S$. The best solution found is the new solution $S'$.

Step 4: Computer $\Delta = C(S') - C(S)$.

Step 5:

- If {(Δ□ 0), then set $S = S'$.
- Otherwise, compute the probability, $p = \exp(-\Delta\, kT)$. Then, generate a random number θ from U[0, 1]; and set $S = S'$ if $\theta \le p$.
- Update $S_b = S'$ *if* { $C(S') - C(S_b)$}.

Step 6: Repeat Steps 3-5, until the number of iterations reach $N$. If the terminating condition $T = T_F$ is met, then stop.Otherwise, let $T = \alpha T$, and go to step3.

## V. ALGORITHM PARAMETER TUNING

The developed simulated annealing algorithm was implemented in Visual Basic for Applications (VBA) and was tested on a Core(TM) i3-3227U processor 1.90GHz with 4.00GB of RAM laptop computer. In order to fine tune the parameters of the algorithm and improve its performance, a standard problem set A from Augerat et al. [9] is selected.

TABLE I
COMPARISON OF RESULTS FOR INSTANCE SET A

| Instance | Best Know Optimal | Best found | CPU Time (Sec.) | % off optimal |
|---|---|---|---|---|
| A-n32-k5 | 784 | 814 | 512 | 3.83 |
| A-n33-k5 | 661 | 662 | 834 | 0.15 |
| A-n33-k6 | 742 | 744 | 845 | 0.27 |
| A-n34-k5 | 778 | 799 | 862 | 2.70 |
| A-n36-k5 | 799 | 834 | 874 | 4.38 |
| A-n37-k5 | 669 | 697 | 1056 | 4.19 |
| A-n37-k6 | 949 | 974 | 1061 | 2.63 |
| A-n38-k5 | 730 | 768 | 1086 | 5.21 |
| A-n39-k5 | 822 | 857 | 1089 | 4.26 |
| A-n39-k6 | 831 | 842 | 1278 | 1.32 |
| A-n44-k7 | 937 | 963 | 1467 | 2.77 |
| A-n45-k6 | 944 | 1032 | 1701 | 9.32 |
| A-n45-k7 | 1146 | 1179 | 1718 | 2.88 |
| A-n46-k7 | 914 | 996 | 1798 | 8.97 |
| A-n48-k7 | 1073 | 1146 | 1945 | 6.80 |
| A-n53-k7 | 1010 | 1130 | 2105 | 11.88 |
| A-n54-k7 | 1167 | 1276 | 2164 | 9.34 |
| A-n55-k9 | 1073 | 1159 | 2250 | 8.01 |
| A-n60-k9 | 1408 | 1470 | 2620 | 4.40 |
| A-n61-k9 | 1035 | 1128 | 2684 | 8.99 |
| A-n62-k8 | 1290 | 1434 | 2509 | 11.16 |
| A-n63-k9 | 1634 | 1704 | 3087 | 4.28 |
| A-n63-k10 | 1315 | 1450 | 3115 | 10.27 |
| A-n64-k9 | 1402 | 1562 | 3135 | 11.41 |
| A-n65-k9 | 1177 | 1373 | 3218 | 16.65 |
| A-n69-k9 | 1168 | 1338 | 3273 | 14.55 |
| A-n80-k10 | 1764 | 1970 | 4425 | 11.68 |
| | | | Average | 6.75 |
| | | | S.D. | 4.42 |
| | | | Min | 0.15 |
| | | | Max | 16.65 |

This benchmark problem set contains problem instances where both customer locations are uniformed scattered around the depot and demands are generated from a uniform distribution. Size of the problem instances ranges from 31 to 79 customers. The best-known solutions have been proved to be the optimal ones for every instance of this benchmark.

After fine tuning, the parameters were set at $T_s = 100$, $T_f = 0.00001$, $\alpha = 0.98$, and $N = 600$. Performance of the algorithms in all 27 problem instances on problem set A is given in Table 1. The average % off-optimal is 6.75 and the SD is 4.42.

## VI. Numerical Example

### *A. The Problem Instance*

Consider a VRPT instance with 20 customers. Each customer has a daily demand that must be satisfied directly from the depot, so-called regular demand; and transshipment demand that can be satisfied from inventory at another customer. The problem instance is to be solved in two consecutive days (Day 1, and Day 2) in order to evaluate the impact of satisfying the transshipment demand. The depot has three delivery vehicles of same type, each with a capacity 100 units. Distance between a pair of customers is estimated from (X, Y) coordinate. Regular demands are randomly generated from a uniform distribution. The customer locations and regular demand data are shown in Table II. In addition, there are three levels of percentage: 5%, 10%, and 20% of customers that require transshipment demand (i.e. one customer, two customers, and four customers). Table 3 provides the transshipment demand data. The problem instance with 5% transshipment demand only contains transshipment demand No.1. The instance with 10% transshipment demand has transshipment demands No. 1 and No. 2. Finally, the 20% transshipment demand instance contains all four transshipment demands (No.1-4).

The problem instance is solved twice. The first is when only regular demands from the depot can be delivered, and transshipment demands are picked up and brought back to the depot. Then, the transshipment demand from Day 1 will be added to the regular demand to be delivered in Day 2. The second time is when both regular and transshipment demands must be delivered on the same day. That is, the generated delivery routes must contain both customer 12 and customer 18 and that customer 12 is visited first.

The difference in the total cost between allowing and not allowing transshipment demand delivery can provide the impact of including transshipment demand in the delivery route.

### *B. Results and Discussion*

Results from numerical example are shown in Table 4. The results indicate that allowing transshipment demand to be delivered on the same day could incur additional cost of 6 (i.e.0.48%) for the case when there is one customer requiring the transshipment demand. A closer look reveals that there is a cost savings of 15 on Day 2, which is not enough savings to offset the additional cost of 21 that incurs on Day 1. This suggests that allowing transshipment delivery could lead to cost savings in some cases, which remains to be investigated further. The same results can be seen for the 10% and 20% transshipment demand, i.e. additional cost of 21 and 10, respectively.

The benefit from allowing same day delivery of transshipment is the reduction in the carryover demand from day-to-day that is caused by having to bring the transshipment demand back to the depot on Day 1 to deliver on Day 2. More importantly, this benefit can be significant from the service level to end customer standpoint. Being able to deliver on the same day implies that the end customer would receive the item faster. This reduction in the lead time is especially important because it is the lead time for the item that was previously unavailable to the end customer, i.e. the very reason of performing transshipment. In other words, it is a tradeoff between a slightly higher cost and better customer service.

TABLE II
LOCATIONS CONSIDERED IN THIS CASE STUDY AND A SET OF REGULAR DEMAND

| **Node** | **Location** | | **Regular Demand** | |
|---|---|---|---|---|
| **Depot** | **X Cord.** | **Y Cord** | **Day1** | **Day2** |
| 0 | 14 | 68 | - | - |
| **Customer** | **X Cord.** | **Y Cord.** | **Day1** | **Day2** |
| 1 | 96 | 44 | 19 | 20 |
| 2 | 50 | 5 | 21 | 18 |
| 3 | 49 | 8 | 6 | 8 |
| 4 | 13 | 7 | 19 | 17 |
| 5 | 29 | 89 | 7 | 9 |
| 6 | 58 | 30 | 12 | 8 |
| 7 | 84 | 39 | 16 | 18 |
| 8 | 14 | 24 | 6 | 9 |
| 9 | 2 | 39 | 16 | 20 |
| 10 | 3 | 82 | 8 | 12 |
| 11 | 5 | 10 | 14 | 11 |
| 12 | 98 | 52 | 21 | 21 |
| 13 | 84 | 25 | 16 | 17 |
| 14 | 61 | 59 | 3 | 5 |
| 15 | 1 | 65 | 22 | 20 |
| 16 | 88 | 61 | 18 | 21 |
| 17 | 9 | 2 | 19 | 20 |
| 18 | 19 | 32 | 1 | 3 |
| 19 | 93 | 3 | 24 | 22 |
| 20 | 50 | 93 | 8 | 8 |

TABLE III
TRANSSHIPMENT DEMAND

| **Transshipment No.** | **Day1** | | |
|---|---|---|---|
| | **Pick-up customer** | **Delivery To** | **Demand** |
| 1 | 12 | 18 | 3 |
| 2 | 2 | 14 | 5 |
| 3 | 19 | 6 | 8 |
| 4 | 15 | 20 | 8 |

TABLE IV
TEST RESULT ON 5%, 10% AND 20% TRANSSHIPMENT

| | **% transshipment** | | | | | |
|---|---|---|---|---|---|---|
| | **5%** | | **10%** | | **20%** | |
| **Allow Transshipment** | **No** | **Yes** | **No** | **Yes** | **No** | **Yes** |
| Day1 | 619 | 640 | 620 | 647 | 622 | 661 |
| Day2 | 637 | 622 | 628 | 622 | 630 | 601 |
| Total cost | 1256 | 1262 | 1248 | 1269 | 1252 | 1262 |
| Difference | | 6 | | 21 | | 10 |
| % Difference | | 0.48 | | 1.68 | | 0.8 |

## VII. CONCLUSIONS

In this article, a simulated annealing algorithm for vehicle routing problem with transshipment has been developed. The VRPT considers both regular demands that are satisfied by the items from the depot and transshipment demands of items that can only be satisfied from other customers. The algorithm developed is based on the simulated annealing algorithm. Parameters of the algorithm are fine-tuned with a standard problem set to improve its performance. A numerical example is provided to demonstrate the VRPT, benefit of allowing same day transshipment delivery, and the performance of the algorithm. Results from preliminary test are promising. A more thorough and extensive computational study will be conducted to clearly show the algorithm performance and benefit of transshipment on various problem sizes and transshipment percentages.

## REFERENCES

[1] G. B. Dantzig and J. H. Ramser, "The Truck Dispatching Problem," *Management Science*, vol. 6, no. 1, pp. 80–91, Oct. 1959.

[2] M. M. Alipour, "A Learning Automata Based Algorithm For Solving Capacitated Vehicle Routing Problem," *IJCSI International Journal of Computer Science Issues*, vol. 9, no. 1, pp. 138–145, 2012.

[3] S. Zhang, C. Zhu, J. K. O. Sin, and P. K. T. Mok, "A novel ultrathin elevated channel low-temperature poly-Si TFT," *IEEE Electron Device Letters*, vol. 20, no. 11, pp. 569–571, Nov. 1999.

[4] M. Gendreau, G. Laporte, and R. Séguin, "Stochastic vehicle routing," *European Journal of Operational Research*, vol. 88, no. 1, pp. 3–12, Jan. 1996.

[5] J. Brandão, "A new tabu search algorithm for the vehicle routing problem with backhauls," *European Journal of Operational Research*, vol. 173, no. 2, pp. 540–555, Sep. 2006.

[6] F. Alfredo Tang Montané and R. D. Galvão, "A tabu search algorithm for the vehicle routing problem with simultaneous pick-up and delivery service," *Computers & Operations Research*, vol. 33, no. 3, pp. 595–619, Mar. 2006.

[7] Y. Nagata, O. Bräysy, and W. Dullaert, "A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows," *Computers & Operations Research*, vol. 37, no. 4, pp. 724–737, Apr. 2010.

[8] F. YANG and H. XIAO, "Models and Algorithms for Vehicle Routing Problem with Transshipment Centers," *Systems Engineering - Theory & Practice*, vol. 27, no. 3, pp. 28–35, Mar. 2007.

[9] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by Simulated Annealing," *Science*, vol. 220, no. 4598, pp. 671–680, May 1983.

[10] P. Augerat, J. M. Belenguer, E. Benavent, A. Corberan, D. Naddef, and G. Rinaldi, "Computational results with a branch and cut code for the capacitated vehicle routing problem," *Istituto di analisi de sistemi ed informatica consiglio nazionale delle ricerche*, no. 495, 1998.