Abstract—The objective of this research is to develop a cost model for the implementation of ANSI/ASQC Z1.4 single acceptance sampling plans in multi-stage inspection processes. The cost model is developed based on the concept of cost of quality. The developed cost model is used to select parameter values n and c of the ANSI/ASQC Z1.4 sampling plans at each inspection step to optimize total cost of quality. The results are that the proposed cost-based sampling plans provide 6.19% cost saving, equivalent to 8,809,611 Baht per year.

Keywords—acceptance sampling plan, cost of quality, cost model, multi-stage inspection

I. INTRODUCTION

Quality inspection is commonly implemented to appraise the quality of product lots. Inspection can be either 100% inspection or sampling-based. Regarding sampling, single acceptance sampling plan is commonly used to decide whether to accept or reject the inspected lot. ANSI/ASQC Z1.4 is a type of acceptance sampling plan that is widely-used. The parameter of this plan is defined by sample size, n and acceptance number, c. To decide on the parameter values for n and c, quality measures such as average outgoing quality (AOQ) are considered [1]. However, utilizing the sampling plan with only the consideration of quality measures does not necessarily yield cost effectiveness. Thus, there is a need to take into account the costs associated with the implementation of sampling plan when deciding the sampling plan parameter values.

Cost of quality has been widely studied since it explains the relationship between quality-based activities and costs. Generally, cost of quality consists of three parts: prevention costs, appraisal costs, and failure costs [2]. Consideration of cost of quality helps in managing activities regarding quality more cost-effective. For example, if companies consider the total cost of quality associated with the use of sampling plan, they are able to find the appropriate parameter values of the sampling plan the minimize the total cost of quality.

The relationship between sampling plan activities and associated costs has been investigated by many researchers. Chakraborty and Bapaye [3] studied the effect of inspection error in using a standard sampling method [MIL STD 105D] on appraisal cost and average outgoing quality limit. Maxim, Cullen and Mardo [4] presented a research paper on sampling plan and appropriate sampling amount in order to arrive at the minimum expenditure level. Many studies have focused on finding the elements of cost of quality. There are many research papers that explain the elements of cost of quality. Appraisal cost is mainly described by inspection cost [4], [5], [6], [7], [8], [9], [10], [11], [12]. Failure costs compose of elements such as scrap cost [4], [6], [8], [10], [11], [12], rework cost [4], [5], [7], cost of defective investigation [4], cost associated with downgraded product [4], customer claim [6], [11], cost of product replacement [8]. Prevention costs compose of cost of quality planning [10], training cost [11], and auditing cost [11].

In order to design the sampling plan parameter values for n and c to yield optimal cost of quality, the elements of cost of quality should be thoroughly considered. However, past studies have not yet taken some cost elements into consideration. Moreover, the detailed methods to obtain those cost elements are not clearly illustrated.

Another issue to consider is that most of past studies considered the design of sampling plan at final inspection stage only. In some real cases, acceptance sampling inspection is performed after each production stage before allowing the lot to subsequent stage. The design of appropriate parameter values for sampling plans of multi-stage inspection should be considered to obtain total cost of quality resulted from all processes. Freeman [10] suggested a method to estimate quality cost of multi-stage inspection. However, this work does not focus on the determination of sampling plan parameters to optimize total quality cost.

This study thus has the objectives to develop the cost model that involves more complete cost elements for the implementation of ANSI/ASQC Z1.4 single acceptance sampling plans in multi-stage inspection processes. Next, the developed cost model is used to select parameter values for n and c of the ANSI/ASQC Z1.4 sampling plans at each of the inspection steps to optimize total cost of quality. However, this study does not consider prevention cost since the selection of the parameter values of the sampling plan does not affect this type of cost.

II. DEVELOPMENT OF COST OF QUALITY MODEL ASSOCIATED WITH MULTI-STAGE INSPECTION PROCESSES

A. Description of Multi-Stage Inspection Processes and Associated Costs

This study uses the production processes of Printed Circuit Board (PCB) to develop the cost model. The
developed cost model is particularly designed for the PCB study processes. However, the concept of the model can be generalized to other cases. The production of PCB product has a process flow shown in Fig. 1.

The process flow can be explained as follows. Each production lot passes through three manufacturing processes: 1) Plate Through Hole (PTH), 2) Panel Plate (PP), and 3) Solder Mask (S/M). There are two main types of defectives that are possible in each process. They are electrical defective (e.g. short or open circuit) and visual defective (e.g. error on physical appearance). Each lot is inspected for only visual defectives at the end of each manufacturing process according to ANSI/ASQC Z1.4 acceptance sampling plans. If the lot is accepted, the lot is then passed to the next process. If the lot is rejected, all units are then 100% re-inspected. Then, accepted units are passed to the next step, while defectives are reworked. The units, which are unable to be reworked, then need to be scrapped. After each lot passes through all three manufacturing processes, it next enters the E-test station, where all units are tested for electrical properties. Last station is visual inspection station, where all units are visually inspected before delivery to the customers. At the customer side, all units are inspected as they are used in the customer’s process. If the customer finds either unacceptable electrical or visual defectives, they will send a claim to the supplier and request the supplier to find out the cause of defectives. The incurred costs at this stage are due to the error of false acceptance of defectives at 100% E-test and 100% visual inspection stages. Defectives found at the customer site can not be reworked.

Currently, the sampling inspection in each process is based on Institute of Printed Circuits (IPC-6012B) sampling method, which is not cost-concerned. This research aims to design the cost-based sampling plans with the implementation of the widely-used ANSI/ASQC Z1.4 standard instead. The measures of success are the reduction of total cost of quality and the better quality level as measured by the average outgoing quality (AOQ).
B. Definition of Related Factors in the Cost Model

In the multi-stage inspection processes, the objective is to find out the combination of sampling plan parameter values (n and c) of all processes that provides the minimum total cost of quality of all processes. The elements of costs in the model are calculated per one lot inspection. The cost model for multi-stage inspection processes is shown in section C, where the definition of related factors in the cost model is described in section B.

Table I: Elements of Costs Occur in Each Process

<table>
<thead>
<tr>
<th>Processes</th>
<th>Cost elements</th>
<th>Cost of inspection per unit</th>
<th>Cost of reworking per unit</th>
<th>Product cost per unit</th>
<th>Cost of scrap destruction per production unit</th>
<th>Penalty cost of defective investigation per unit</th>
<th>Cost of defective investigation per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTH</td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>S/M</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>E-Test</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Visual Inspection</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Customer</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

There are three types of quality costs, which are the results of the lot inspection activities. They are inspection cost, cost of lot rejection, and cost of lot acceptance. Cost of lot rejection means the costs associated with activities after rejecting the lot. It comprises of elements, which are cost of reworking defectives, product cost of scrapped units, and cost of scrap destruction. Cost of lot acceptance means the costs associated with activities after finding defectives that have been accepted as good units in previous processes. Cost of accepting defectives comprises of product cost of scrapped units, penalty cost, and cost of defective investigation. The cost of lot rejection and the cost of lot acceptance are failure costs. TABLE I shows the relationship of processes, types of quality costs, and cost elements.

B. Definition of Related Factors in the Cost Model

\[ P_{a_{i}} = \text{Probability of lot acceptance at process } i \]
\[ P_{r_{i}} = \text{Probability of lot rejection at process } i \]
\[ P_{v_{i}} = \text{Proportion of defectives that can be reworked at process } i \]
\[ P_{e_{i}} = \text{Proportion of defectives that can not be reworked at process } i \]
\[ P_{v_{i}} = \text{Proportion of visual defectives at process } i \]
\[ P_{e_{i}} = \text{Proportion of electrical defectives at process } i \]
\[ P_{ve_{i}} = \text{Proportion of defectives due to both electrical and visual properties at process } i \]
\[ P_{vo_{i}} = \text{Proportion of visual defectives after passing inspection process } i \]
\[ P_{eo_{i}} = \text{Proportion of electrical defectives after passing inspection process } i \]
\[ P_{ev_{i}} = \text{Proportion of defectives due to both electrical and visual properties after passing inspection process } i \]
\[ P_{eav_{i}} = \text{Probability of error of false rejection at 100% visual inspection process} \]
\[ P_{ece_{i}} = \text{Probability of error of false rejection at 100% electrical inspection process} \]
\[ P_{evo_{i}} = \text{Probability of error of false acceptance at 100% visual inspection process} \]
\[ P_{ebe_{i}} = \text{Probability of error of false acceptance at 100% electrical inspection process} \]
\[ c_{i} = \text{Inspection cost per unit at process } i \]
\[ c_{i} = \text{Rework cost per unit at process } i \]
\[ c_{i} = \text{Product cost per unit at process } i \]
\[ c_{i} = \text{Cost of scrap destruction per unit at process } i \]
\[ c_{i} = \text{Penalty cost per unit at process } i \]
\[ c_{i} = \text{Cost of defective investigation per unit at process } i \]
\[ T_{i} = \text{Inspection time per unit at process } i \text{ (hr/unit)} \]
\[ S_{i} = \text{Inspector wage at process } i \text{ (Baht/hr)} \]
\[ M_{i} = \text{Cost of inspection machine and tools at process } i \]
\[ S_{i} = \text{Salvage value of inspection machine at process } i \]
\[ Y_{i} = \text{Life of inspection machine at process } i \text{ (year)} \]
\[ V_{i} = \text{Number of units inspected per month} \]
\[ L_{d_{i}} = \text{Direct labor cost per unit at process } i \]
\[ L_{d_{i}} = \text{Indirect labor cost per unit at process } i \]
\[ M_{i} = \text{Direct material cost per unit at process } i \]
\[ M_{i} = \text{Indirect material cost per unit at process } i \]
\[ L_{d_{i}} = \text{Direct labor cost of rework per unit at process } i \]
\[ L_{d_{i}} = \text{Indirect material cost of rework per unit at process } i \]
\[ T_{c} = \text{Delivery cost for returned units per month} \]
\[ \text{Se} = \text{Inspector wage for defective investigation per month} \]
\[ V_{c} = \text{Number of defectives delivered to customer per month} \]
C. Cost of Quality Model Associated with Multi-Stage Acceptance Sampling Plans

Total cost of quality can be written as shown in Equation 1.

$$E(C) = \sum_{i=1}^{m} \{E(C_{i}) + E(C_{r}) + E(C_{a_{i}})\}$$

(1)

where

- $i = 1$, PTH manufacturing process
- $i = 2$, PP manufacturing process
- $i = 3$, S/M manufacturing process
- $i = 4$, 100% E-test process
- $i = 5$, 100% visual inspection process
- $i = 6$, 100% inspection process at customer site

The equations for calculating each of the cost types in each process are shown in the following equations:

**Inspection Cost, $E(C_{i})$**

For $i = 1, 2, 3$

$$E(C_{i}) = ATI_{i}(ci)$$

(2)

$$ATI_{i} = n + \frac{P_{\pi}}{P_{\pi}}(N - n)$$

(3)

For $i = 4, 5$

$$E(C_{i}) = N_{i}(ci)$$

(4)

**Cost of Lot Rejection, $E(C_{r})$**

For $i = 1, 2, 3$

$$E(C_{r}) = P_{\pi_{i}} \left[ N_{i} \cdot P_{m_{i}} \cdot P_{n_{i}} \cdot (cw_{i} + (cx_{i} + cd_{i})) \right]$$

(5)

$$E(C_{r}) = N_{4} \left[ P_{w_{m}} \cdot (1 - P_{w_{b_{m}}}) \right] + P_{w_{m}} \cdot (1 - P_{w_{b_{m}}}) \cdot N_{5}$$

(6)

$$E(C_{r}) = N_{6} \left[ P_{w_{m}} \cdot (1 - P_{w_{b_{m}}}) \right] + P_{w_{m}} \cdot (1 - P_{w_{b_{m}}}) \cdot N_{6}$$

(7)

Cost of Lot Acceptance, $E(C_{a_{i}})$

$$E(C_{a_{i}}) = N_{6} \cdot \left( P_{w_{m}} + P_{w_{b_{m}}} - P_{w_{b_{m}}} \right) \cdot (cc_{s} + ct_{b} + cs_{b} + cd_{b})$$

(8)

The probability of defectives that are passed to the next process after the inspection at each process can be calculated from the concept of average outgoing quality (AOQ) [1]. The equations can be written as follows:

For $i = 1$

$$P_{v_{m}} = \frac{P_{n_{i}} \cdot P_{m_{i}} \cdot N_{i} - P_{n_{i}} \cdot n_{i}}{N_{2}}$$

(9)

$$P_{v_{m}} = \frac{P_{n_{i}} \cdot P_{m_{i}} \cdot N_{i} - P_{n_{i}} \cdot P_{n_{i}}} {N_{2} + \frac{P_{n_{i}} \cdot N_{i} - P_{n_{i}} \cdot P_{n_{i}}}{N_{2}}}$$

(10)

For $i = 2, 3$

$$P_{v_{m}} = \frac{P_{n_{i}} \cdot P_{m_{i}} \cdot N_{i} - P_{n_{i}} \cdot n_{i}}{N_{2}}$$

(11)

$$P_{v_{m}} = \frac{P_{n_{i}} \cdot P_{m_{i}} \cdot N_{i} - P_{n_{i}} \cdot n_{i}}{N_{2}}$$

(12)

The comparative results of the cost-based sampling plans and the current non cost-based IPC-6012B sampling plan are shown in TABLE III. The results are compared in terms of total cost of quality and the quality
level as defined by average outgoing quality (AOQ). The AOQs are shown at two stages: after passing the last sampling inspection process (AOQ3) and after passing the final 100% visual inspection process (AOQ5). AOQ5 represents the proportion of defectives delivered to the customer. The results are compared under two scenarios: with penalty cost and without penalty cost. In the PCB case study, there is a penalty cost of 10 times of product price for returned defectives. In some other cases, there is no penalty cost but there is only cost of replacement. It can be seen from TABLE III that under the penalty cost of 10 times of product price, the optimal cost-based sampling plans are obtained by using the minimum acceptance number of one for all processes \([(n,c) = (32,1)]\). These plans are the strictest plans as possible to mitigate the cost of penalty due to defectives. The total cost of quality under the optimal cost-based sampling plans is 35,470 Baht per lot versus 37,810 Baht per lot under the non cost-based IPC-6012B plan. The saving is 6.19% or 8,809,611 Baht per year. The quality levels AOQ3 and AOQ5 when using the cost-based sampling plans are 0.0835 and 0.0028 respectively. These numbers are lower than those of 0.1089 and 0.0050 under the use of non cost-based IPC-6012B sampling plan. These numbers indicate that the obtained quality level when using the cost-based sampling plans is higher that of the current IPC-6012B sampling plans.

### TABLE II : EQUATIONS FOR CALCULATING CONSTANTS IN COST ELEMENTS

<table>
<thead>
<tr>
<th>Item</th>
<th>With penalty cost</th>
<th>Without penalty cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost-based ANSI/ASQC Z1.4 sampling plans</td>
<td>IPC-6012B</td>
</tr>
<tr>
<td></td>
<td>(n, c) of PTH process</td>
<td>(32,1)</td>
</tr>
<tr>
<td></td>
<td>(n, c) of PP process</td>
<td>(32,1)</td>
</tr>
<tr>
<td></td>
<td>(n, c) of SM process</td>
<td>(32,1)</td>
</tr>
<tr>
<td>Total inspection cost (baht per lot)</td>
<td>7,634</td>
<td>7,244</td>
</tr>
<tr>
<td>Total cost of lot rejection (baht per lot)</td>
<td>24,519</td>
<td>24,651</td>
</tr>
<tr>
<td>Total cost of lot acceptance (baht per lot)</td>
<td>3,318</td>
<td>5,915</td>
</tr>
<tr>
<td>Total Cost of Quality (baht per lot)</td>
<td>35,470</td>
<td>37,810</td>
</tr>
<tr>
<td>AOQ5</td>
<td>0.0035</td>
<td>0.1089</td>
</tr>
<tr>
<td>AOQ3</td>
<td>0.0028</td>
<td>0.0050</td>
</tr>
</tbody>
</table>

Comparing the parameter values and costs when there is a high penalty cost and without penalty cost, when there is no penalty cost, the inspection can be less stringent. It can be seen from TABLE III that the acceptance number of PP process can be increased to 6 when there is no penalty cost as compared to the stringent value of 1 when there is a penalty cost. All types of costs are lower. The inspection cost under no penalty scenario is 7,363 Baht per lot, which is lower than that of 7,634 Baht per lot with penalty. The reason is that with less stringent plan, the probability of lot rejection is lower. Then, the number of units to be inspected after lot rejection is less, causing lower cost of inspection. Regarding the cost of lot rejection, since the probability of lot rejection is lower with the less stringent plan, thus the number of rejected parts is lower. This fact results in the lower costs of rework and scrap when there is no penalty cost as can be seen by the cost of lot rejection of 23,811 Baht per lot versus 24,519 Baht per lot with high penalty cost scenario. Regarding the cost of lot acceptance, it can be easily explained that this type of cost is lower when there is no penalty cost (857 Baht per lot versus 3,318 Baht per lot) since the penalty cost is a main element of cost of lot acceptance.

### CONCLUSION

This research has the objective to develop the cost model for the implementation of ANSI/ASQC Z1.4 single acceptance sampling plans in multi-stage inspection processes. The cost model was developed based on the concept of cost of quality. The developed cost model was used to select parameter values for \(n\) and \(c\) of the ANSI/ASQC Z1.4 sampling plans at each of the inspection steps to optimize total cost of quality. This research suggests the classification of costs associated with the decision of sampling plans to three types: inspection cost, cost of lot rejection, and cost of lot acceptance. The equations to calculate each of the cost elements were presented. The results are that the proposed cost-based sampling plans provide lower total cost of quality inspection.
cost of quality and better quality level than those of the current non cost-based sampling plans. The cost reduction of 6.19% is obtained that is equivalent to cost saving of 8,809,611 Baht per year.

REFERENCES